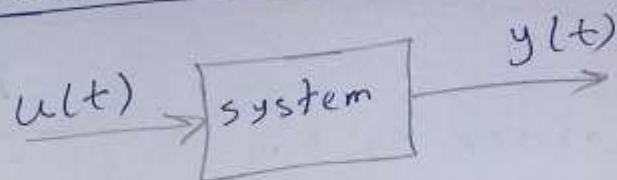


Lec 18

→ state-space representation:-



Mathematical model

① Differential equation:-

$$y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \dots + a_0 y(t) = b_m u^{(m)}(t) + \dots + b_0 u(t)$$

② Transfer Function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}$$

$n \rightarrow$ system order

③ block diagram.

④ signal flow graph.

⑤ state-space model.

SISO

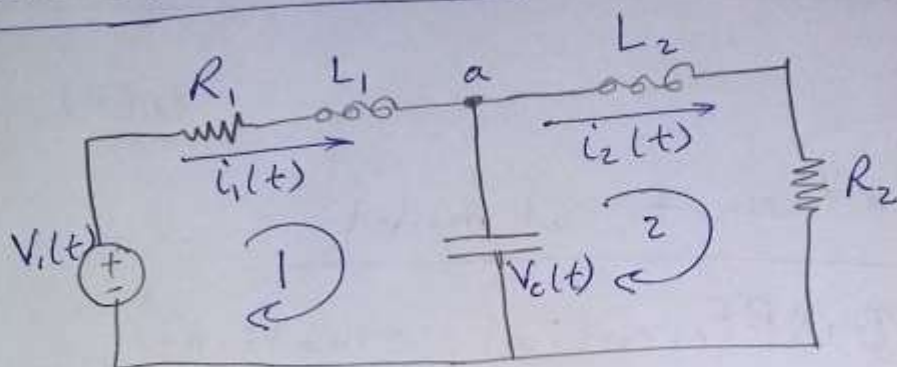
$$\dot{x}(t)_{n \times 1} = A_{n \times n} x(t)_{n \times 1} + B_{n \times 1} u(t)_{1 \times 1}$$

$$y(t)_{1 \times 1} = C_{1 \times n} x(t)_{n \times 1} + D_{1 \times 1} u(t)_{1 \times 1}$$

$n \rightarrow$ number of states variable.

Ex Find state space model

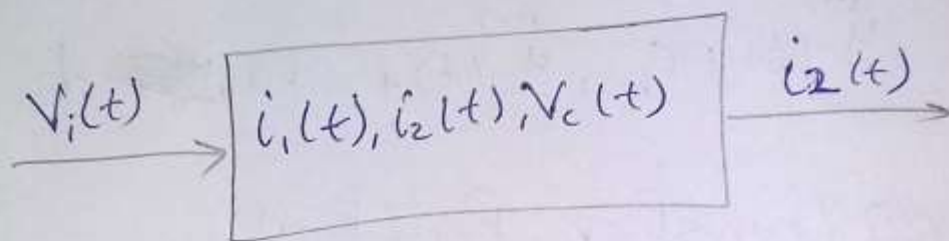
3rd order system



$$x_1(t) = i_1(t)$$

$$x_2(t) = i_2(t)$$

$$x_3(t) = V_c(t)$$



$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + D u(t)$$

$$\dot{X}_1(t) = \frac{d}{dt} i_1(t)$$

KVL at loop (1)

$$V_i(t) = R_1 i_1(t) + L_1 \frac{d}{dt} i_1(t) + V_c(t)$$

$$\dot{X}_1(t) = \frac{-R_1}{L_1} X_1(t) - \frac{1}{L_1} X_3(t) + \frac{1}{L_1} u(t) \rightarrow (1)$$

$$\dot{X}_2(t) = \frac{d}{dt} i_2(t)$$

→ Apply KVL at loop 2:-

$$V_c(t) + L_2 \frac{d}{dt} i_2(t) + i_2 R_2 = 0$$

$$\dot{X}_2(t) = \frac{-R_2}{L_2} X_2(t) - \frac{1}{L_2} X_3(t) \rightarrow (2)$$

$$\dot{X}_3(t) = \frac{d}{dt} V_c(t)$$

* apply KCL at node (a)

$$i_1(t) = C \frac{dV_c(t)}{dt} + i_2(t)$$

$$\dot{X}_3(t) = \frac{1}{C} X_1(t) - \frac{1}{C} X_2(t) \rightarrow (3)$$

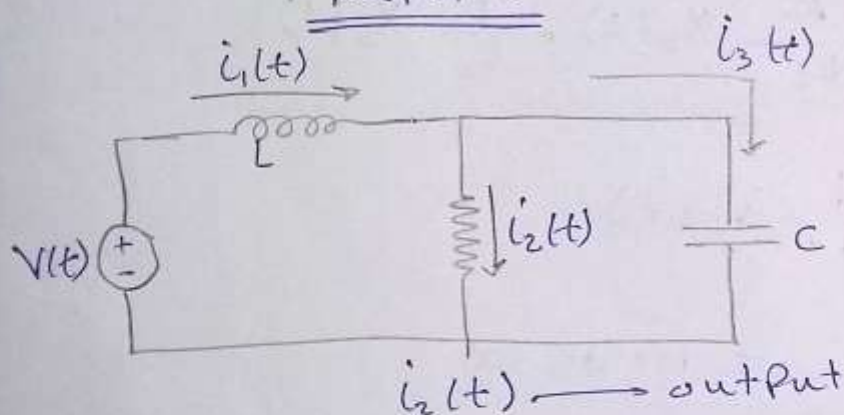
$$y(t) = i_2(t) \Rightarrow y(t) = x_2(t)$$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & -\frac{1}{L_2} \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} + \begin{pmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{pmatrix} u(t)$$

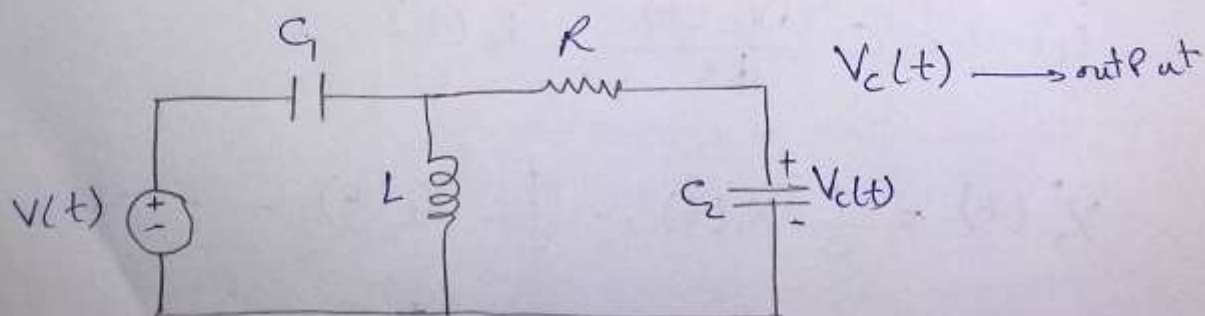
$$y(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} + D \underline{u(t)}$$

Report

[1]



[2]



[4] Lec 18

ex

Given

$$G(s) = \frac{5s^2 + 8s + 15}{s^3 + 4s^2 + 10s + 20}$$

Required

$$\dot{X}(t) = A X(t) + B u(t)$$

$$y(t) = C X(t) + D u(t)$$

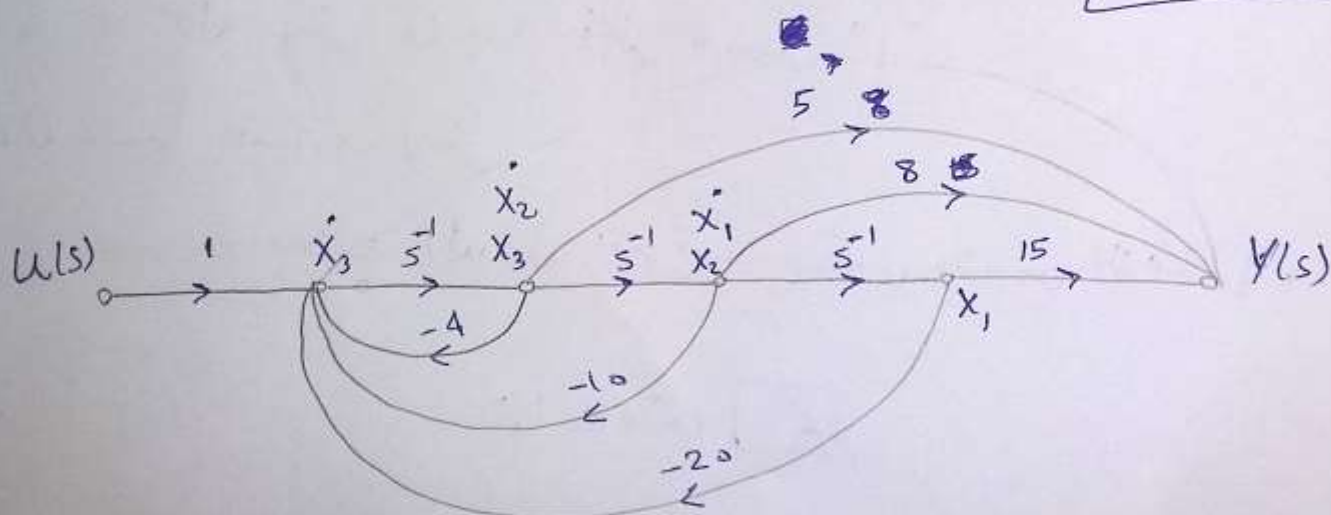
→ Draw signal flow graph :-

① نقسم البسط، البسط إلى 3 أجزاء 3 أجزاء

$$G(s) = \frac{5s^{-1} + 8s^{-2} + 15s^{-3}}{1 + 4s^{-1} + 10s^{-2} + 20s^{-3}} \quad \left. \vphantom{\frac{5s^{-1} + 8s^{-2} + 15s^{-3}}{1 + 4s^{-1} + 10s^{-2} + 20s^{-3}}} \right\} \text{integral form}$$

$$s \frac{5s^{-1} + 8s^{-2} + 15s^{-3}}{1 - [-4s^{-1} - 10s^{-2} - 20s^{-3}]}$$

CCF



"state diagram

Σ F

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -20 & -10 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 15 & 8 & 5 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + D u(t)$$

$$G(s) = \frac{5s^2 + 8s + 15}{s^3 + 4s^2 + 10s + 20}$$

← اذا طلب (ccf) تكتب المعنوفات مباشرة

A ← أول صف يكون (0 1 0) ثم ننقل "1" إلى اليمين (Right Shift)

← آخر صف هو معاملات المقام بترتيب معكوس وإشارات معكوسة.

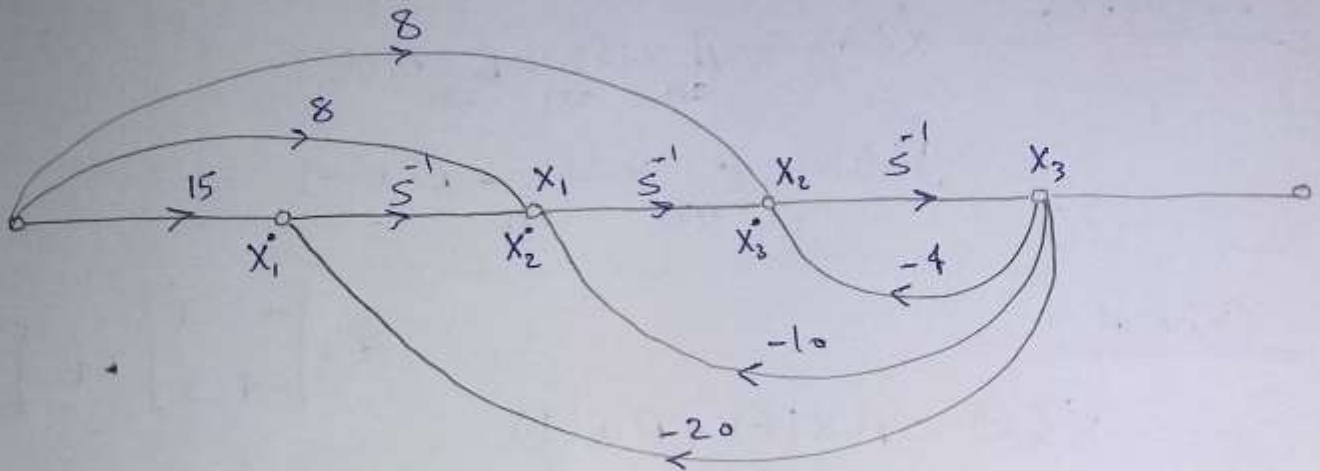
B ← كله صفر ما عدا آخر عنصر "1".

D ← يساري صفر.

C ← معاملات البسط بترتيب عكسي بنفس الإشارات.

6 Lec 18

— توجد طريقة أخرى تسمى (o.c.f)



$$\begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \\ \dot{X}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -20 \\ 1 & 0 & -10 \\ 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix} + \begin{bmatrix} 15 \\ 8 \\ 5 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix} + D \cdot u(t)$$

$$A_c = A_o^T$$

$$A_o = A_c^T$$

$$B_c = B_o^T$$

$$B_o = C_o^T$$

$$C_o = B_o^T$$

$$C_c = B_c^T$$

Given $\rightarrow G(s) = \frac{Y(s)}{U(s)} = \frac{10}{s^2 + 3s + 4}$

required $\dot{x}(t) = \underset{2 \times 2}{A} \underset{2 \times 1}{x(t)} + \underset{2 \times 1}{B} u(t)$

$y(t) = \underset{1 \times 2}{C} \underset{2 \times 1}{x(t)} + \underset{1 \times 1}{D} u(t)$

Given

$A = \begin{bmatrix} 0 & 1 \\ -4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\dot{x}(t) = A x(t) + B u(t)$

$y(t) = C x(t) + D u(t)$

$C = [1 \ 0]$, $D = 0$

Required $G(s) = \frac{Y(s)}{U(s)} = C [sI - A]^{-1} B + D$

Sol

$sI - A = s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 4 & s+3 \end{bmatrix}$

$[sI - A]^{-1} = \begin{bmatrix} s+3 & 1 \\ -4 & s \end{bmatrix} \times \frac{1}{\begin{vmatrix} s & -1 \\ 4 & s+3 \end{vmatrix}}$

$= \begin{bmatrix} s+3 & 1 \\ -4 & s \end{bmatrix} \times \frac{1}{s^2 + 3s + 4}$

$$C(sI - A)^{-1}B + D = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{s^2 + 3s + 4} \begin{bmatrix} s+3 & 1 \\ 4 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2 + 3s + 4} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$G(s) = \frac{10}{s^2 + 3s + 4}$$

Report

$$G(s) = \frac{Y(s)}{U(s)} = \frac{8s^3 + 40s^2 + 50s + 60}{s^4 + s^3 + 20s^2 + 10s + 30}$$

* Draw state diagram in c.c.f and o.c.f

* Draw state space in c.c.f and o.c.f

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